RESEARCH ARTICLE Am. J. PharmTech Res. 2012; 2(2) ISSN: 2249-3387

AMERICAN JOURNAL OF PHARMTECH RESEARCH

Journal home page: http://www.ajptr.com/

Performance Study on Capillary-Tissue Diffusion Phenomena for Blood Flow through Stenosed Blood Vessels

Sapna Ratan Shah

1. *Department of Mathematics, Harcourt Butler Technological Institute, Kanpur - 208002, (India)*

ABSTRACT

The study focuses on the behavior of diffusion phenomenon in the normal and stenosed capillary-tissue exchange system where the rheology of flowing blood in the capillary is characterized by the generalized Bingham Plastic fluid model. Assessment of the severity of the disease could be made possible through the variation of a parameter named as retention parameter. The concentration profile and associated physiological diffusion variable involved in the study for normal and diseased state have been analyzed. The model is also employed to study the effect of shape of stenosis on flow characteristics. An extensive quantitative analysis is performed through numerical computations of the desired quantities having physiological relevance through their graphical representations so as to validate the applicability of the present model.

Keywords: Bingham Plastic fluid model, Capillary-tissue exchange, Resistance to flow, Wall shear stress, Stenosis shape parameter.

*Corresponding Author Email: sapna1980jan@rediffmail.com

Received 16 March 2012, Accepted 28 March 2012

Please cite this article in press as: Shah SR. et al., Performance Study on Capillary-Tissue Diffusion Phenomena for Blood Flow through Stenosed Blood Vessels . American Journal of PharmTech Research 2012.

INTRODUCTION

Stenosis is formed by substances depositing on vessel walls. A stenosis may lead to partial or total vessel blockage in some instances and therefore poses a serious medical problem[1, 2]. The actual reason for formation of stenosis "Atherosclerotic" (Figure.1) is not known, but its effect over the flow characteristics has been studied by many research workers, [3,4,5,6]. Flow and diffusion through capillary-tissue exchange system has been identified as one of the thrust areas of research. In narrow capillaries, at times, the arterial transport become much larger as compared to axial transport and it contributes to the development of atherosclerotic plaques, greatly reducing the capillary diameter.

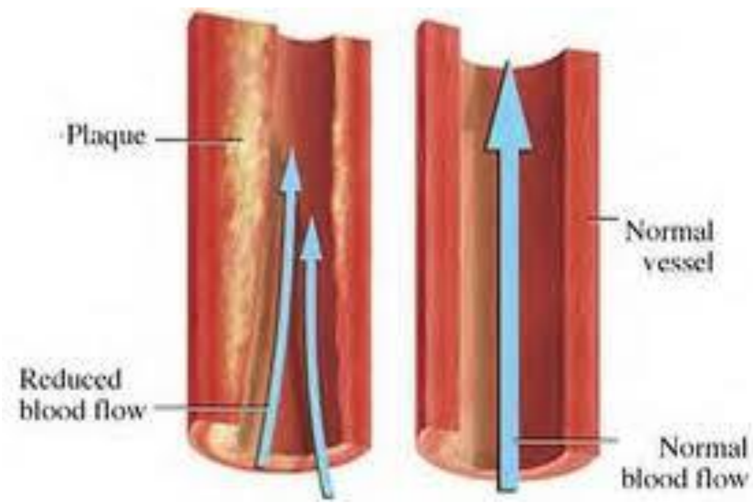

Figure.1. Atherosclerotic

The problem of flow and diffusion become much more difficult through a capillary with stenosis at some region. The response of blood flow through an artery under stenotic conditions has been attempted by[7,8] . Accordingly, considerable effort has been expended studying the fluid mechanics of flow through a stenosis[9,10,11]. Several workers[12,13,14] proposed various representative models for blood in narrow capillaries. Viscosity depending on the local variation of the concentration of the suspended cells has been introduced by Sugihara[15]. Secomb[16] studied the effect of concentration on viscosity and the effect of the concentration on blood flow through a vessel with stenosis and found it an important aspect from physiological point of view. Cristini[17] have also discussed the effect of the variation of concentration of the suspended cells of blood. The theoretical study of Yakhot[18] pointed out that blood obeys the Casson's equation only in the limited range, except at very high and very low shear rate and that there is no difference between the Casson's plots and Bingham-plastic's plots of experimental data. In this paper capillary tissue diffusion phenomenon of blood flow has been investigated and the effect of stenosis on the resistance to flow, apparent viscosity and wall shear stress in an artery by considering the blood as a Bingham Plastic fluid is also obtained. To examine the effect of stenosis shape parameter, blood flow is considered through an axially non-symmetrical but radially symmetric stenosis such that the axial shape of the stenosis can be change just by varying a parameter, stenosis shape parameter (m).

Formulation of the problem:

Considering the axisymmetric laminar steady flow of blood, the general constitutive equation in the case of a mild stenosis subject to the additional conditions, may therefore be written as:

$$-\frac{\partial P}{\partial r} + \frac{1}{r}\frac{\partial (r\tau)}{\partial z} = 0, \qquad -\frac{\partial P}{\partial r} = 0, \tag{1}$$

Where p is the fluid pressure, (- ∂p / ∂z) is pressure gradient in artery, where (z, r) are (axial, radial) co-ordinates with z measured along the axis and r measured normal to the axis of the artery, and (P,τ) are (Pressure, Shear stress).

The concentration equation for the solute is expressed by

$$u\frac{\partial C}{\partial z} = D\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r}\right) \tag{2}$$

Where C represents the concentration of the solute, u is the axial velocity and D the diffusion coefficient for the solute under consideration in the blood.

2.1 Boundary conditions:

To solve the above system of equations, the following boundary conditions are introduced:

$$\begin{aligned} &\frac{\partial u}{\partial r} = 0 && \text{at } r = 0 \\ &u = 0 && \text{at } r = R(z) \\ &P = P_0 && \text{at } z = 0 \\ &P = P_L && \text{at } z = L \\ &\frac{\partial C}{\partial r} = 0 && \text{at } r = 0 \\ &D\frac{\partial C}{\partial r} = VNC && \text{at } r = R \end{aligned} \tag{3}$$

Where N is retention parameter, C is concentration; u is the axial velocity and D the diffusion coefficient.

2.2 Bingham plastic fluid model:

For Bingham plastic fluid, the stress-strain relation is given by

$$\tau = \tau_0 + \mu\left(-\frac{du}{dr}\right) \qquad (4)$$

where $\tau = \left(-\frac{dp}{dz}\frac{r}{2}\right)$, $\tau_0 = \left(-\frac{dp}{dz}\frac{R_P}{2}\right)$,

u : axial velocity

μ : viscosity of fluid

(-dp/dz) :pressure gradient

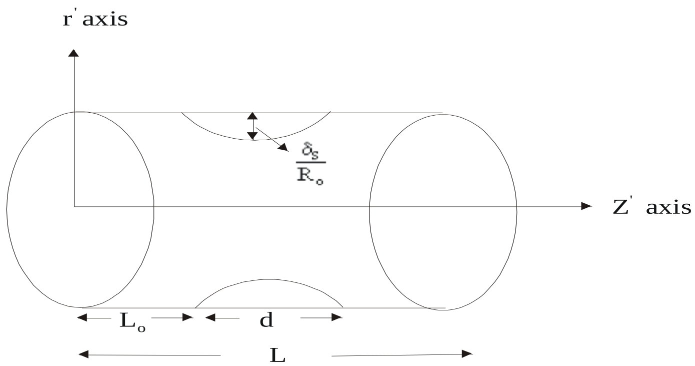

Figure.2. Stenotic blood vessel

The stenosis studied in the present study is shown in figure.2. It is assumed that the stenosis develops in the capillary wall in an axially non-symmetric but radially symmetric manner and depends upon the axial distance z and the height of its growth.

$$\frac{R(z)}{R_0} = 1 - A[L_0^{(m-1)}(z-d)-(z-d)^m], \qquad d \le z \le d + L_0 \qquad (5)$$

$$= 1, \qquad \text{otherwise,}$$

Where the parameter $A = \frac{\delta}{R_0 L_0^m}\frac{m^{m/(m-1)}}{(m-1)}$

and δ denotes the maximum height of stenosis at $z = (d + L_0 / m^{m/(m-1)})$. The ratio of the stenosis height to the radius of the normal artery is much less than unity. R(z) and R_0 are the radius of the artery with and without stenosis respectively. L_0 is the stenosis length, d represents the location of stenosis and m is stenosis shape parameter. In the case of $m \ge 2$, stenosis shape parameter indicates an axially symmetric stenosis.

Solution of the problem:

The expression for the velocity, u obtained as the solution of equation (1) subject to the boundary conditions (3) and equation (4), is obtained as (for $R_P \le r \le R(z)$)

$$u = -\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2\right] + \frac{\tau_0 R_0}{\mu}\left[\left(\frac{R}{R_0}\right) - \left(\frac{r}{R_0}\right)\right] - \frac{4R_0^{3/2}\tau_0}{3\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)^{1/2}\left[\left(\frac{R}{R_0}\right)^{3/2} - \left(\frac{r}{R_0}\right)^{3/2}\right] \qquad (6)$$

The constant plug flow velocity, u_P may be obtained from equation (5) evaluated at $r = R_P$.

The volumetric flow rate Q can be defined as,

$$Q = \int_0^R 2\pi u r\, dr = \pi \int_0^R r\left(-\frac{du}{dr}\right) dr, \tag{7}$$

The flow flux, Q when Rp<< R (i.e., the radius of the plug flow region is very small as compared to the non-plug flow region), is calculated as

$$Q = -\frac{R_0^4 \pi}{8\mu}\frac{dp}{dz}\left(\frac{R}{R_0}\right)^4 +\!- \frac{\tau_0 \pi}{3\mu}\left(\frac{R}{R_0}\right)^3 +\!- \frac{4R_0^{7/2}\pi}{7}\left\{\frac{\tau_0}{\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)\left(\frac{R}{R_0}\right)^7\right\}^{1/2} \tag{8}$$

$$Q = \frac{\pi R^4}{8\mu}\left(-\frac{dp}{dz}\right) f(\bar{y}), \tag{9}$$

From above equation pressure gradient is written as follows,

$$\left(-\frac{dp}{dz}\right) = \frac{8\mu Q}{\pi R_0^4} f(\bar{y}) \tag{10}$$

$$f(\bar{y}) = \left(\bar{y}\right)^4 +\!- \frac{\tau_0 \pi}{3\mu}\left(\bar{y}\right)^3 +\!- \frac{4R_0^{7/2}\pi}{7}\left\{\frac{\tau_0}{\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)\left(\bar{y}\right)^7\right\}$$

Integrating equation (10) using the condition (3) $P = P_0$ at $z = 0$ and $P = P_L$ at $z = L$. We have

$$\Delta P = P_L - P_0 = \frac{8\mu Q}{\pi R_0^4}\int_0^L \frac{dz}{(R(z)/R_0)^4 f(\bar{y}(z))} \tag{11}$$

The resistance to flow is denoted by λ and defined as follows,

$$\lambda = \frac{P_L - P_0}{Q} \tag{12}$$

The resistance to flow from equation (11) using equations (12) is written as,

$$\lambda = 1 - \left(L_0/L\right) + \left(f_0/L\right)\int_d^{d+L_0} \frac{dz}{(R(z)/R_0)^4 f(\bar{y}(z))} \tag{13}$$

where f_0 is given by $f_0 = (R/R_0)^4 +\!- \frac{\tau_0 \pi}{3\mu}(R/R_0)^3 +\!- \frac{4R_0^{7/2}\pi}{7}\left\{\frac{\tau_0}{\mu}\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)(R/R_0)^7\right\}$

Following the apparent viscosity (μ_{app}) is defined as follows;

$$\mu_{app} = \frac{1}{\left(R(z)/R_0\right)^4 f(\bar{y})} \tag{14}$$

The shearing stress at the wall can be defined as;

$$\tau_R = \tau_0 + \mu\left(-\frac{du}{dr}\right)_{r=R(z)} \tag{15}$$

To solve the Eq. (4) takes the form:

$$\frac{\nu R_0^2}{D_1 L}\frac{\partial C_1}{\partial x} = \frac{\partial^2 C_1}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial C_1}{\partial \eta} \tag{16}$$

The boundary conditions are:

$$\frac{\partial C_1}{\partial \eta} = 0 \qquad \text{at} \quad \eta = 0,$$
$$D_1\frac{\partial C_1}{\partial \eta} = VNC_1 \qquad \text{at} \quad \eta = \frac{R}{R_0} \tag{17}$$

$$u = -\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[\eta^2\frac{\tau_0 R_0}{\mu}\left(\frac{R'}{R_0}\right)^{1/2} - \left(\frac{r}{R_0}\right)^{2/3}\eta\right] + \frac{5R_0\tau_0}{8\mu}\left[\eta\left(\frac{R'}{R_0}\right) - \left(\frac{r}{R_0}\right)\right]$$
$$-\frac{6R_0^{3/2}\tau_0}{7\mu}\eta\left(-\frac{1}{2\mu}\frac{dp}{dz}\right)^{1/2}\left[\frac{3R_0\tau_0}{2\mu}\eta\left(\frac{R'}{R_0}\right)^{3/2} - \eta\left(\frac{r}{R_0}\right)^{3/2}\right] + \left[\eta^2\left(\frac{R'}{R_0}\right) - \left(\frac{r}{R_0}\right)^{1/2}\eta\right] \tag{18}$$

On using Eq. (17) the solution for concentration subject to the boundary conditions is given as:

$$C_1 = \frac{R_0^{\ 3}}{4\mu L^2 D_1}\left(-\frac{dp}{dx}\right)\left(\frac{\partial C_1}{\partial x}\right)\frac{\bar{u}}{5 D_1 L^5}\eta_c^2\left[\frac{R'\eta_c^2}{2} + \frac{R'\eta^3}{5} - \frac{R_c^2\eta}{3}\right]$$
$$[\left(\frac{R'\eta^3}{5}\right) - \left(\frac{\eta^4 8}{3}\left(\frac{R^5\eta^2}{5} - \frac{54\eta^{5/2}}{6}\right)\right) + 2\eta_c^2\left(\frac{R'\eta_c^2}{3} - \frac{R_c^4\eta^2}{5}\right)]$$
$$-\left(\frac{R'\eta^4}{5}\frac{\partial C_1}{\partial z}\frac{\bar{u}}{5D_1 L^5}\eta_c^2\right) + \frac{R'\eta^3}{5}\left(\frac{R'\eta_c^2}{2} - \frac{R_c^2\eta}{3}\right) + M \tag{19}$$

Where

$$M = [\frac{R_0^4}{4\mu L^2}\left(-\frac{dp}{dz}\right)\{\frac{R'^2}{4}\left(1 - \frac{\eta_c}{R'^3}\frac{VNR_0R'}{D_1(2n+1)}\frac{\eta_c}{R'^3}\left(1 - \frac{V^2R_0}{D_1}\right)\right)^2 - \frac{8}{D^2{}_1 N}\eta_c^2 R\left(\frac{5}{\eta_c} - \frac{VNR_0R'}{D_1}\right)$$
$$+\frac{\eta_c}{R'^3}\left(1 - \frac{V^2R_0}{D_1}\right) - \frac{R'R_0^2}{L}\bar{u}\left(1 - \frac{\eta_c}{R'^3}\frac{V^2NR_0R'}{D_1}\right) + \left(\frac{5}{\eta_c} - \frac{VNR_0R'}{D_1}\right)\}]$$

RESULTS AND DISCUSSION:

In order to have estimate of the quantitative effects of various parameters involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow, concentration profile and associated physiological diffusion variables for normal and diseased system associated with stenosis due to the local deposition of lipids have been determine. The results are shown in Figure 3-8 by using the values of parameter based on experimental data in capillary. Figures show the results for resistance to flow for different values of stenosis shape parameter, stenosis length, and stenosis size. Figur.3 reveals the variation of resistance to flow (λ) with stenosis size (δ/R_0) for different values of stenosis shape parameter (m). It is observed that the resistance to flow (λ) decreases as stenosis shape parameter (m) increases and maximum resistance to flow (λ) occurs at (m = 2), i. e. in case of symmetric stenosis. It has also been seen from this graph that resistance to flow (λ) increases as stenosis size (δ/R_0) increases. Resistance to flow increase as stenosis grows or radius of artery decreases. This referred to as Fahraeus-Lindquist effect in very thin tubes. The present results are therefore consistent with the observations[5,4,15].

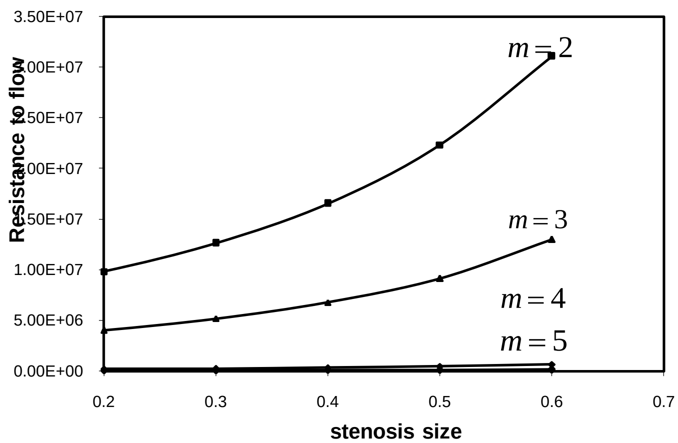

Figure.3 Variation of resistance to flow with stenosis size for different Values of stenosis shape parameter

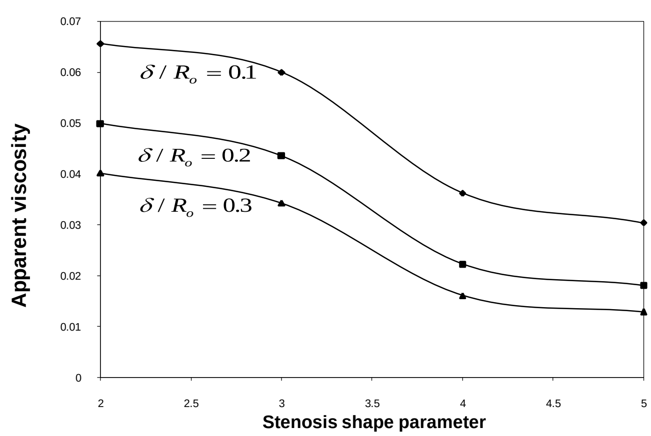

Figure.4 Variation of apparent viscosity with stenosis shape parameter for different values of stenosis size

Figure.4 shows the variation of apparent viscosity for different values of stenosis shape parameter. It is observed that the apparent viscosity decreases as stenosis shape parameter increases. It may also observe from the figure that apparent viscosity increases as stenosis size (δ/R_0) increases. The results are compared with[15,16,17,18]. It is clear that apparent viscosity increases as stenosis grows. But the same is not true in the absence of stenosis.

In capillary flow, the viscosity of blood flow found to vary with the radius of the capillary. The development of stenosis accelerates the velocity of plasma between the cells. This in turn increases the concentration of red cell and viscosity of blood in stenotic region, therefore increases. Figure.5 shows the variation of wall shear stress (τ) with stenosis size for different values of stenosis shape parameter. It is clear from the figure that the wall shear stress (τ) increases as stenosis size increases. It has also been seen from this graph that the wall shear stress (τ) decreases as stenosis shape parameter (m) increases and maximum resistance to flow (λ) occurs at (m = 2), i. e. in case of symmetric stenosis. These results are consistent to the observation of (12).

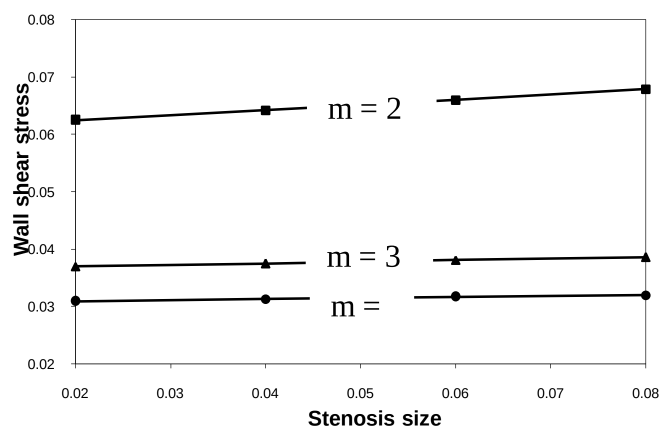

Figure.5 Variation of wall shear stress with stenosis size for different values of stenosis shape parameter

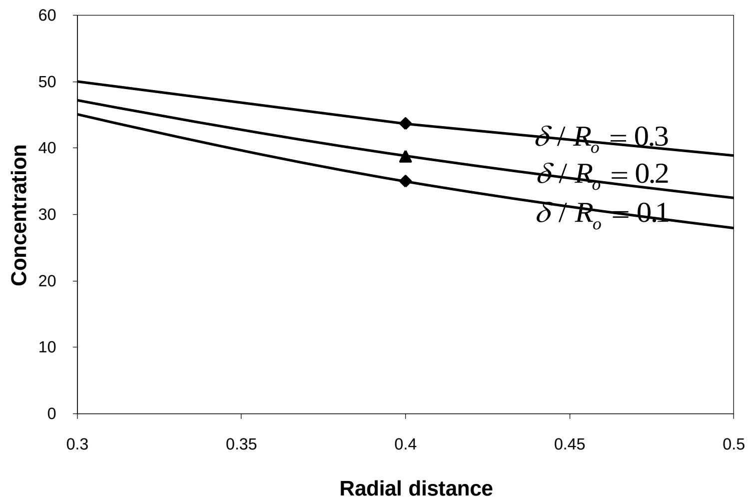

Figure.6 Variation of concentration with radial distance for different values of stenosis size

Figure.6 shows the diffusion of large and small molecular weight nutrients within the capillary region for different values of stenosis size. Large molecular weight nutrients within the capillary region face more resistance to diffuse into the tissue and therefore the cells of the deeper region are deprived of getting sufficient nutrition. This result is consistent with result of1[5, 18]. Figure.7 represents the effects of retention parameter (N) on concentration in blood flow capillary region. Increasing values of retention parameter described the increase in retention of solute within the blood flow in the capillary region. The value of retention parameter (N=1) implies the complete retention. No solute or fluid diffuses and as retention parameter decreases from 1 to 0.4 more solute diffuses, which in turns, decreases the solute concentration in the capillary region. The variation of the values of retention parameter in the stenotic region may also be associated with the type of plaques deposited on the walls: calcified, fibrous or fatty plaque.

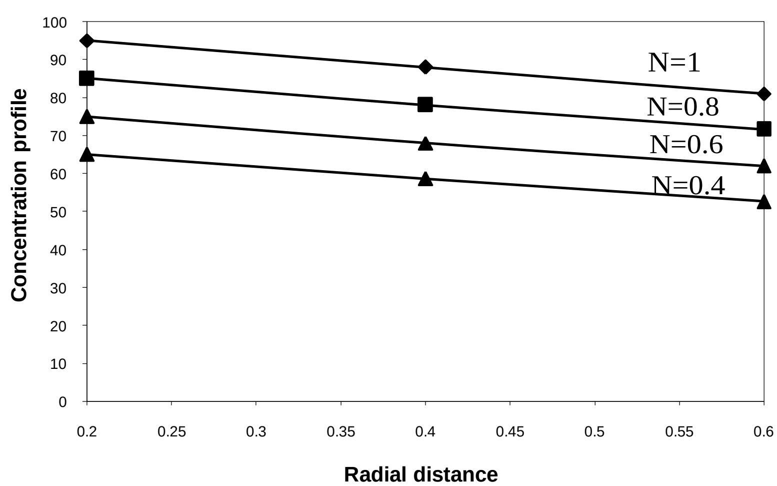

Figure.7 Variation of concentration with radial distance for different Values of retention parameter

CONCLUSION

The present study incorporates the more realistic representation for blood in small diameter blood vessels and simultaneous dispersion of solute in capillary in normal and stenotic depending on various parameters including retention parameter. Bingham Plastic fluid model appears to be realistic in the sense that the equations are fairly closely to the blood flow and the central core region is easily represented. The results are more encouraging and correlating well with the experimental observation that deeper region cells are deprived of the nutrients in the stenotic region. The results also indicate that as the degree of the stenosis area severity increases, the pressure gradient required to impel the blood passing through the narrowing channel increases significantly.

REFERENCES:

1 Biswas D Chakarborty US Pulsatile flow of blood in a constricted artery with body acceleration. J Applic and Applied Mathe 2009; 2: 329-342.

2 H Mogens. An Evaluation of a Micropolar A Harry Model for Blood Flow through an Idealized Stenosis J Biomech 1989; 22: 21-218.

3 Johnston B Johnston PR Corney S Kilpatrick D Non-Newtonian blood flow in human right coronary arteries: Steady state simulation J Biomech 2004 37: 709-720.

4 K Haldar Blood flow and red blood cell deformation Math Biol 1985 47545.

5 Mandal KP Chakravarty S Mandal A Amin N Effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosesd artery. J Applied Mathematics and Computation 2007: 189; 766-779.

6 Misra JC Adhikar SD Shit GC Mathematical analysis of blood flow through an arterial segment with time dependent stenosis. J Math Model and Analy 2008: 13, 401-412.

7 Nagarani P Sarojamma G Effect of body acceleration on pulsatile flow of casson fluid through a mild stenosed artery J Korea-Australia Reo 2008: 20, 189-196.

8 Quarteroni A Tuveri M Veneziani A Computational vascular fluid dynamics; Problems, models and methods Compu. Visuali in Science: 2000; 2:163-197.

9 Sanjeev KA mathematical model for Newtonian and non-Newtonian flow through tapered tubes Int Review Pure and Applied Mathematics 2009:5.

10 Sanjeev K Archana D Effect of porous parameter for the blood flow in a time dependent stenotic artery. In J Biomech Special Issue 2009; 7-8.

11 Sankar DS Ismail AI Two-fluid mathematical models for blood flow in stenosed artery Boundary Value Problems 2009; 1-15.

12 Sankar DS Lee U Two-fluid Casson model for pulsatile blood flow through stenosed arteries. Communi in Non-linear Sci & Numer Simul 2009.

13 Shankar DS Hemalatha K Pulsatile flow of Herschel-Bulkley fluid through stenosed arteries A mathematical model. Int J Non-linear Mechanics, 2006; 41, 979- 990.

14 Sharan M Popel AS A two-phase model for flow of blood in tubes with increased effective viscosity near the wall. Biorheo 2001; 38. 415-428.

15 Sugihara M Motion of a sphere in a cylindrical tube filled with a Brinkman medium. Fluid Dyn Res 2003; 34, 59-76.

16 Secomb TW Pries AR Blood flow and red blood cell deformation in non-uniform capillaries: effect of endothelial surface layer. Microc 2005; 9, 189-196.

17 Cristini V Ghassan S Computer Modeling of red blood cell Rheology in the Microcirculation A Brief Overview AMBE 2005; 33; 1-4.

18 Yakhot A Grinberg L Nikitin N Modeling rough stenoses by an immersed-boundary method J Biomech 2004; 3: 78-89.